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**MATHEMATICS**

**SPECIALIST**

**UNIT 1**

**Semester One**

**2018**

**SOLUTIONS**

***Calculator−free Solutions***

1. (a) $b-a=\left(\begin{matrix}1\\3\end{matrix}\right)-\left(\begin{matrix}2\\1\end{matrix}\right)=\left(\begin{matrix}-1\\2\end{matrix}\right)$✓

 $∴a∙\left(b-a\right)=\left(\begin{matrix}2\\1\end{matrix}\right)∙\left(\begin{matrix}-1\\2\end{matrix}\right)=-2+2=0$ ✓✓

 $∴a⊥\left(b-a\right)$

 (b) (i) $\vec{OC}=\vec{AB}=b-a=-i+2j$✓

(ii) $\vec{AC}=\vec{OC}-\vec{OA}=-3i+j$✓

 $∴\left|\vec{AC}\right|=\sqrt{10}$ ✓

 OR

 $\left|\vec{AC}\right|=\left|\vec{OB}\right|$ ✓

 $∴\left|\vec{AC}\right|=\left|\begin{matrix}1\\3\end{matrix}\right|=\sqrt{10}$ ✓ [6]

2. (a) (i) $\frac{13!+12!}{13!-12!}=\frac{13×12!+12!}{13×12!-12!}=\frac{12!\left(13+1\right)}{12!\left(13-1\right)}$ ✓

 $=\frac{14}{12}=\frac{7}{6}$ ✓

 (ii) $\frac{\_{4}}{} =\frac{10!}{4!×6!}÷\frac{8!}{4!×4!}$ ✓

 $=\frac{10×9×8!×4!×4!}{8!×6×5×4!}=\frac{10}{5}×\frac{9}{6}$ $=2×\frac{3}{2}=3$ ✓

 (b) LHS $=k\left(\begin{matrix}n\\k\end{matrix}\right)=k×\frac{n!}{k!\left(n-k\right)!}$ ✓

 $=\frac{k}{k}×\frac{n!}{\left(k-1\right)!\left[n-k\right]!}$ ✓

 $=n×\frac{\left(n-1\right)!}{\left(k-1\right)!\left[\left(n-1\right)-(k-1)\right]!}$ ✓✓

 $=n\left(\begin{matrix}n-1\\k-1\end{matrix}\right)=$ RHS (Or from RHS to LHS) [8]

3. (a) $-3>-5$ but $\left(-3\right)^{2}=9>\left(-5\right)^{2}=25$ is false ✓

 (b) “If the triangle is not equilateral, then the triangle does not

 have three equal sides.” ✓

 Yes is it always true since the original implication is always

 true by definition of equilateral triangles. ✓

 (c) “If n is divisible by 3, then n is divisible by 6”. ✓

 The converse is not always true, because for n to be

 divisible by 6 is must be divisible by both 2 and 3. ✓

 (d) FOR ALL natural numbers p, EXISTS a real number q, ✓✓

 such that q is the square root of p. [7]

4. (a) $a+b=\left(\begin{matrix}2\\-1\end{matrix}\right)+\left(\begin{matrix}-1\\4\end{matrix}\right)=\left(\begin{matrix}1\\3\end{matrix}\right)$✓

 $∴\hat{\left(a+b\right)}=\pm \frac{1}{\sqrt{10}}\left(\begin{matrix}1\\3\end{matrix}\right)$ ✓

 $∴u=2\sqrt{10}×\pm \frac{1}{\sqrt{10}}\left(\begin{matrix}1\\3\end{matrix}\right)=\pm \left(\begin{matrix}2\\6\end{matrix}\right)$ ✓



✓

 (b) (i) $c=ka$✓

 $\left(\begin{matrix}-4\\α\end{matrix}\right)=k\left(\begin{matrix}2\\-1\end{matrix}\right) \rightarrow k=-2 \rightarrow α=2$ ✓

 (ii) $\left|\left(\begin{matrix}-4\\α\end{matrix}\right)-\left(\begin{matrix}-1\\4\end{matrix}\right)\right|=3\left|\begin{matrix}2\\-1\end{matrix}\right|$

 $∴\sqrt{9+\left(α-4\right)^{2}}=3\sqrt{5}$ ✓

 $9+\left(α-4\right)^{2}=9×5$ ✓

 $\left(α-4\right)^{2}=36$

 $α=4\pm 6 \rightarrow α=10 or -2$ ✓✓ [10]

5. (a) $\vec{AC}∙\vec{OB}=\left(c-a\right)∙\left(a+c\right)=0$ ✓

 $c∙a+c∙c-a∙a-a∙c=0$ ✓

 $\left|c\right|^{2}-\left|a\right|^{2}=0 \rightarrow \left|a\right|=\left|c\right|$ ✓

 $∴$ OABC is a rhombus

 (b) LHS $=\left|AC\right|^{2}+\left|OB\right|^{2}=\left|c-a\right|^{2}+\left|a+c\right|^{2}$ ✓

 $=\left(c-a\right)∙\left(c-a\right)+\left(a+c\right)∙\left(a+c\right)$ ✓

 $=c∙c-2a∙c+a∙a+a∙a+2a∙c+c∙c$✓

$=2\left|a\right|^{2}+2\left|c\right|^{2}$✓

$=\left|OA\right|^{2}+\left|AB\right|^{2}+\left|BC\right|^{2}+\left|OC\right|^{2}$ as required [7]

6. (a) ∠AED = 90° ✓

 Triangle in a semi-circle is always right angled. ✓

 (b) ∠ABE = ∠ADE = 60° ✓

 Angles within the same segment are congruent. ✓

 (c) ∠CAE = 80° ✓

 Opposite angles in a cyclic quadrilateral are supplementary ✓

 (d) ∠TCE = ∠CBE = 100° ✓

 The alternate segment theorem. ✓ [8]

7. $\vec{AB}=\frac{2}{3}\vec{AC}$ ✓

 $∴b-a=\frac{2}{3}\left(c-a\right)$

 $\left(\begin{matrix}1\\3\end{matrix}\right)-\left(\begin{matrix}x\\-1\end{matrix}\right)=\frac{2}{3}\left(\begin{matrix}4-x\\y+1\end{matrix}\right)$ ✓

 $∴x=-5$ and $y=5$ ✓✓ [4]

***Calculator−assumed Solutions***

8. (a) Assume all non-repeated numbers are selected from

 both sets: 3, 8, 4, 6 = 4 digits✓

 Plus all remaining digits from one set: 1, 2, 5, 7 = 4 digits

 Plus one more digit to make the first repetition

 $∴$ 4 + 4 + 1 = 9 digits minimum ✓

 (b) Assume the largest numbers are chosen first: ✓

 8 + 7 + 7 + 6 = 28 ✓

 one more digit could include the number 5, making

 the sum over 30.

 $∴$ 4 digits max ✓ [54

9. (a) (i) $=319 770$ ✓

 (ii) $×=103 950$ ✓✓

 (iii) $-=281 010$ ✓✓

 (iv) $×+×=38 760+125 970=164 730$ ✓✓

 (b) (i) $8!=40 320$ ✓

 (ii) $3!×6!=4 320$ ✓✓

 (iii) $8!-2!×7!=30 240$ ✓✓ [12]

10. (a) II and III ✓✓

 (b) $××$ OR $×××6!$ ✓✓✓

 (c) LHS $=\frac{n!}{r!×\left(n-r\right)!}+\frac{n!}{\left(r+1\right)!×\left(n-r-1\right)!}$ ✓

 $=\frac{n!}{r!×\left(n-r-1\right)!}×\left[\frac{1}{\left(n-r\right)}+\frac{1}{\left(r+1\right)}\right]$ ✓

 $=\frac{n!}{r!×\left(n-r-1\right)!}×\left[\frac{r+1+n-r}{\left(n-r\right)\left(r+1\right)}\right]$

 $=\frac{n!×\left(n+1\right)}{r!×\left(r+1\right)×\left(n-r-1\right)!×\left(n-r\right)}$ ✓

 $=\frac{\left(n+1\right)!}{\left(r+1\right)!×\left(n-r\right)!}$ ✓

 $=\frac{\left(n+1\right)!}{\left(r+1\right)!×\left[\left(n+1\right)-\left(r+1\right)\right]!}$ ✓

 $==$ RHS [10]

11. (a) (i) $p+q$ ✓

 (ii) $2p-q$ ✓

 (iii) $-\left(3p+2q\right)$ ✓

 (iv) $2q-p$ ✓

 (b)

✓✓

 [6]

12. (a) $\vec{AC}=\left(\begin{matrix}2\\k\end{matrix}\right)-\left(\begin{matrix}-2\\0\end{matrix}\right)=\left(\begin{matrix}4\\k\end{matrix}\right)=4i+kj$ ✓

 $\vec{BC}=\left(\begin{matrix}2\\k\end{matrix}\right)-\left(\begin{matrix}1\\-3\end{matrix}\right)=\left(\begin{matrix}1\\k+3\end{matrix}\right)=i+\left(k+3\right)j$✓

 (b) $\left|\begin{matrix}4\\k\end{matrix}\right|=\left|\begin{matrix}1\\k+3\end{matrix}\right|$

 $∴\sqrt{4^{2}+k^{2}}=\sqrt{1^{2}+\left(k+3\right)^{2}}$ ✓

 $∴k=1$ ✓

 (c) $D=\left(\frac{-2+1}{2},\frac{0-3}{2}\right)=\left(-0.5,-1.5\right)$ ✓

 (d) $\vec{DC}=\left(\begin{matrix}2\\1\end{matrix}\right)-\left(\begin{matrix}-0.5\\-1.5\end{matrix}\right)=\left(\begin{matrix}2.5\\2.5\end{matrix}\right)$

 $\vec{DB}=\left(\begin{matrix}1\\-3\end{matrix}\right)-\left(\begin{matrix}-0.5\\-1.5\end{matrix}\right)=\left(\begin{matrix}1.5\\-1.5\end{matrix}\right)$ ✓

 $∴\vec{DC}∙\vec{DB}=\left(\begin{matrix}2.5\\2.5\end{matrix}\right)∙\left(\begin{matrix}1.5\\-1.5\end{matrix}\right)=2.5×1.5-2.5×1.5=0$ ✓

 $∴\vec{DC}⊥\vec{DB} \rightarrow $∠CDB is right angled [7]

13. (a) (i) True. ✓

 If a number is divisible by 6, then it is also divisible

 by both 2 and 3. ✓

 (ii) False. ✓

 It must be divisible by both 2 and 3. ✓

 (iii) True. ✓

 The conjunction AND means that it is divisible by both

 2 and 3, and therefore it is also divisible by 6. ✓

 (b) Assume that $n$ is odd AND that $3n+5$ is also odd ✓

 $∴∃ k\in N$ such that $n=2k+1$ ✓

 $∴3n+5$ $=3\left(2k+1\right)+5$ ✓

 $=6k+8=2\left(3k+4\right)=$ even ✓

 Since $3n+5$ is both odd and even simultaneously, this is

 a contradiction, implying that $n$ must be even. ✓

 (c) $\vec{OE}+\vec{AD}+\vec{BF}$

 $=\left[b+\frac{1}{2}\left(a-b\right)\right]+\left[-a+\frac{1}{2}b\right]+\left[-b+\frac{1}{2}a\right]$ ✓✓✓

 $=\left(\frac{1}{2}a-a+\frac{1}{2}a\right)+\left(b-\frac{1}{2}b-\frac{1}{2}b\right)=0$✓ [15]

14. (a) $F\_{1}\cos(30°)=F\_{2}\cos(45°)$ ✓

 $∴\frac{\sqrt{3}}{2}F\_{1}=\frac{1}{\sqrt{2}}F\_{2}$

 $F\_{1}\sin(30°)+F\_{2}\sin(45°)=250$ ✓

 $∴\frac{1}{2}F\_{1}+\frac{1}{\sqrt{2}}F\_{2}=250$

 (b) $F\_{2}=\frac{\sqrt{6}}{2}F\_{1}$

 $∴\frac{1}{2}F\_{1}+\frac{\sqrt{3}}{2}F\_{1}=250$ ✓✓

 $∴F\_{1}=\frac{500}{1+\sqrt{3}}=183.01N$

 $∴F\_{2}=\frac{\sqrt{6}}{2}×\frac{500}{1+\sqrt{3}}=\frac{250\sqrt{6}}{1+\sqrt{3}}=224.14N$ ✓✓

 (c) If $F\_{1}=200N$ then $F\_{2}=\frac{\sqrt{6}}{2}×200=244.95N>200N$ ✓

 $∴$ Cable 2 exceeds its maximum load, hence Cable 1

 must not reach its 200N maximum rating ✓

 If $F\_{2}=200N$ then $F\_{1}=\frac{2}{\sqrt{6}}×200=163.30N<200N$ ✓✓

 Max Force $=F\_{1}\sin(30°)+F\_{2}\sin(45°)$ ✓

 $=\frac{400}{\sqrt{6}}×\frac{1}{2}+200×\frac{1}{\sqrt{2}}=223.07N$ ✓ [12]

15. (a) $n\left(D∪C\right)=n\left(D\right)+n\left(C\right)-n(D∩C)$ ✓

 $810=400+500-n(D∩C)$ ✓

 $∴n\left(D∩C\right)=90$ ✓

 (b) $n\left(D∪C∪B\right)=n\left(D\right)+n\left(C\right)+n\left(B\right)$

 $-n\left(D∩C\right)-n\left(D∩B\right)-n(C∩B)$

 $+n(D∩C∩B)$ ✓

 $900=400+500+210-90-60-110+n\left(D∩C∩B\right)$ ✓

 $∴n\left(D∩C∩B\right)=50$ ✓ [6]

16. (a) $\left(\right)^{2}=+$ ✓

 $\left(\right)^{2}=+$ ✓

 (b) $a=2$ ✓

 $b=n+1$ ✓

 (c) $\left(\right)^{2}=-$ ✓✓

 (d) $\left(\right)^{2}=-$ ✓✓ [8]

17. (a) $\vec{OE}=a+\frac{1}{2}\left(b-a\right)=\frac{1}{2}\left(a+b\right)$ ✓

 $\vec{OF}=c+\frac{1}{2}\left(b-c\right)=\frac{1}{2}\left(b+c\right)$ ✓

 (b) $\vec{DE}=\vec{OE}-\vec{OD}=\frac{1}{2}\left(a+b\right)-\frac{1}{2}a=\frac{1}{2}b$ ✓

 $\vec{GF}=\vec{OF}-\vec{OG}=\frac{1}{2}\left(b+c\right)-\frac{1}{2}c=\frac{1}{2}b=\vec{DE}$ ✓

 $\vec{DG}=\vec{OG}-\vec{OD}=\frac{1}{2}c-\frac{1}{2}a=\frac{1}{2}\left(c-a\right)$ ✓

 $\vec{EF}=\vec{OF}-\vec{OE}=\frac{1}{2}\left(b+c\right)-\frac{1}{2}\left(a+b\right)=\frac{1}{2}\left(c-a\right)=\vec{DG}$ ✓

 $∴\vec{DE}=\vec{GF} and \vec{DG}=\vec{EF} ⇒ $DEFG is a parallelogram ✓ [7]

18. $BF×FD=CF×FA$ ✓

 $∴x\left(2y\right)=2x\left(6\right) \rightarrow y=6 cm$ ✓

 $MD×MB=MT^{2}$ ✓

 $∴4×\left(4+12+x\right)=\sqrt{76}^{2} \rightarrow x=3 cm$ ✓

 $NT^{2}=NC×\left(NC+FC+FA\right)$ ✓

 $∴z^{2}=6×\left(6+6+6\right) \rightarrow z=6\sqrt{3} cm$ ✓ [6]

19. Let $\hat{n}$ be a unit vector perpendicular to $b$

 then, $a=u+\left|u\right|\tan(60°)\hat{n}$✓

 $\left|u\right|=\left|\begin{matrix}3\\1.5\end{matrix}\right|=\frac{3}{2}\sqrt{5}$ ✓

 $n∙\left(\begin{matrix}4\\2\end{matrix}\right)=0 \rightarrow $ let $n=\pm \left(\begin{matrix}-1\\2\end{matrix}\right)$ ✓

 $∴\hat{n}=\pm \frac{1}{\sqrt{5}}\left(\begin{matrix}-1\\2\end{matrix}\right)$ ✓

 $∴a=\left(\begin{matrix}3\\1.5\end{matrix}\right)+\frac{3}{2}\sqrt{5}×\sqrt{3}×\pm \frac{1}{\sqrt{5}}\left(\begin{matrix}-1\\2\end{matrix}\right)=\left(\begin{matrix}3\\1.5\end{matrix}\right)\pm \frac{3}{2}\sqrt{3}\left(\begin{matrix}-1\\2\end{matrix}\right)$

 $∴x=3\pm \frac{3}{2}\sqrt{3}$ and $y=\frac{3}{2}\mp 3\sqrt{3}$ ✓✓ [6]